ASYMPTOTICS OF RESONANT TUNNELING IN QUANTUM WAVEGUIDES WITH SEVERAL EQUAL RESONATORS

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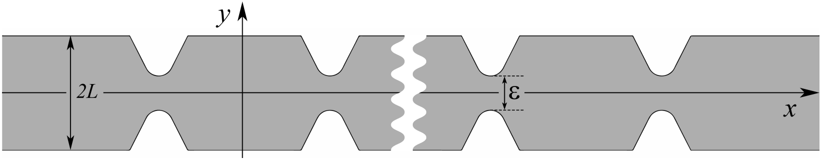


Figure: Waveguide $G(\varepsilon)$

The wave function u of a free electron of energy k^2 satisfies the boundary value problem

$$\left(-\Delta - k^2\right)u(x,y;\varepsilon) = 0, \text{ in } G(\varepsilon);$$
$$u(x,y;\varepsilon) = 0, \text{ on } \partial G(\varepsilon).$$

Radiation conditions

Auxiliary boundary value problem in the cross-section of the strip

$$\left.\begin{array}{c}\left(-\dfrac{\partial^2}{\partial y^2}-\lambda^2\right)\Psi(y)=0, \quad y\in(-L,L);\\ \Psi(-L)=\Psi(L)=0.\end{array}\right| \Rightarrow \left|\begin{array}{c}\lambda_q^2=\left(\dfrac{\pi q}{2L}\right)^2,\\ q=1,2,\dots.\end{array}\right.$$

We assume that

$$\lambda_1^2<k^2<\lambda_2^2.$$

And set the radiation conditions

$$u(x,y;\varepsilon)=\begin{cases}e^{i\nu x}\Psi_1(y)+S_{11}(k;\varepsilon)e^{-i\nu x}\Psi_1(y)+O(e^{\delta x}), & x\to-\infty;\\ S_{12}(k;\varepsilon)e^{i\nu x}\Psi_1(y)+O(e^{-\delta x}), & x\to+\infty;\end{cases}$$

$$\delta>0\,,\qquad \nu=\sqrt{k^2-\lambda_1^2}\,,\qquad \Psi_1(y)=\sqrt{\frac{1}{2L\nu}}\cos\frac{\pi y}{2L}.$$

Resonant tunneling

Reflection coefficient and transition coefficient

$$R(k;\varepsilon) = \left|S_{11}(k;\varepsilon)\right|^2, \qquad T(k;\varepsilon) = \left|S_{12}(k;\varepsilon)\right|^2.$$

We have

$$R(k;\varepsilon) + T(k;\varepsilon) = 1.$$

We are interested in

1. The asymptotics of resonant energies $k_r^2(\varepsilon)$ as $\varepsilon \to 0$;
2. The width $\Upsilon(\varepsilon)$ of resonant peaks at half-height;
3. The heights $T(k_r,\varepsilon)$ of resonant peaks.

Single resonator - reference

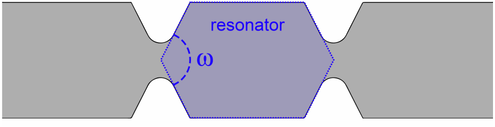


Figure: Waveguide $G(\varepsilon)$ with a single resonator

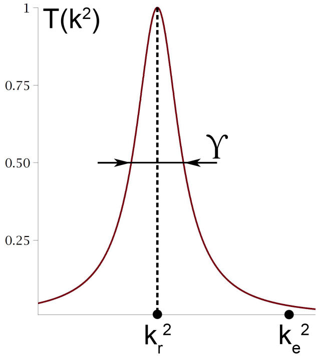
L. Baskin, P. Neittaanmäki, B. Plamenevskii, and O. Sarafanov. *Resonant Tunneling: Quantum Waveguides of Variable Cross-Section, Asymptotics, Numerics, and Applications* // Lecture Notes on Numerical Methods in Engineering and Sciences, Springer, 2015, 275 pp.

The results obtained:

$$k_r^2(\varepsilon) = k_e^2 - Q\varepsilon^{2\pi/\omega} + O(\varepsilon^{2\pi/\omega+2-\delta}),$$

$$T(k;\varepsilon) = \frac{1}{1 + P^2\left(\dfrac{k^2 - k_r^2}{\varepsilon^{4\pi/\omega}}\right)^2}\left(1 + O(\varepsilon^{2-\delta})\right),$$

$$\Upsilon(\varepsilon) = \frac{2}{P}\varepsilon^{4\pi/\omega}\left(1 + O(\varepsilon^{2-\delta})\right).$$



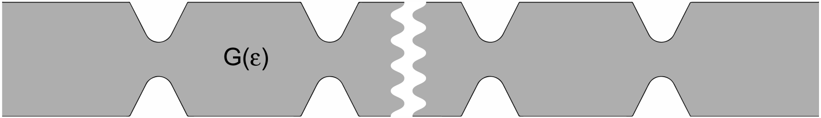
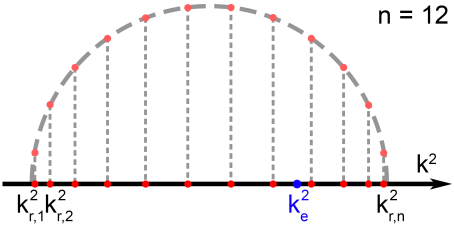


Figure: Waveguide $G(\varepsilon)$ with n equal resonators

The resonant energies

$$k_{r,j}^2(\varepsilon) = k_e^2 - Q_j \varepsilon^{2\pi/\omega} + O(\varepsilon^{2\pi/\omega+2-\delta}), \quad j = 1, \dots, n;$$

$$Q_j = A + B\cos\frac{\pi j}{n+1}.$$

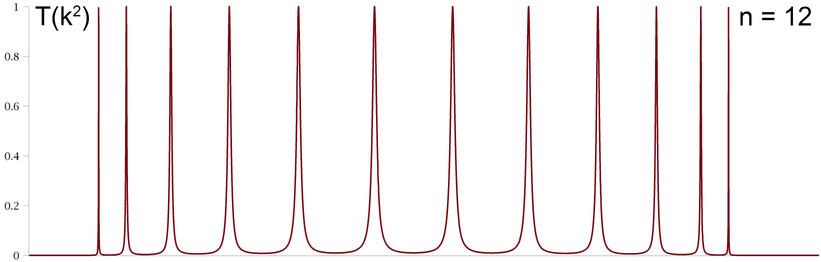


n equal resonators - peaks

The transition coefficient and the widths of peaks

$$T(k;\varepsilon) = \frac{1}{1 + P^2 \left(\frac{\prod_{j=1}^{n}(k^2 - k_{r,j}^2)}{\varepsilon^{2(n+1)\pi/\omega}} \right)^2} \left(1 + O(\varepsilon^{2-\delta})\right),$$

$$\Upsilon_j(\varepsilon) = \frac{4}{P(n+1)} \left(\sin \frac{\pi j}{n+1} \right)^2 \varepsilon^{4\pi/\omega} \left(1 + O(\varepsilon^{2-\delta})\right).$$



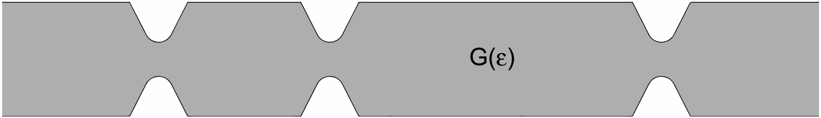


Figure: Waveguide $G(\varepsilon)$ with two different resonators

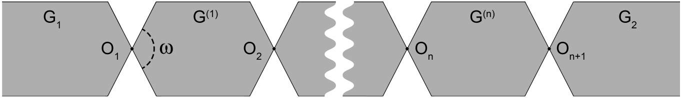
The results obtained

$$k_{r,j}^2 = k_{e,j}^2 - A_j \varepsilon^{2\pi/\omega} + O(\varepsilon^{2\pi/\omega+2-\delta}), \quad j = 1, 2;$$

$$T(k;\varepsilon) = \frac{C_j \varepsilon^{4\pi/\omega}}{1 + P_j^2 \left(\frac{k^2 - k_{r,j}^2}{\varepsilon^{4\pi/\omega}} \right)^2} \left(1 + O(\varepsilon^{2-\delta})\right);$$

$$\Upsilon_j(\varepsilon) = \frac{2}{P_j} \varepsilon^{4\pi/\omega} \left(1 + O(\varepsilon^{2-\delta})\right).$$

Asymptotic of the wave function



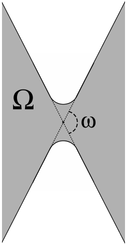


Figure: The "limit waveguide" $G(0)$ (up)
and the limit domain Ω for the problem on w_i (right)

$$u(x,y;\varepsilon) = \chi_{1,\varepsilon}(x,y)v_1(x,y;\varepsilon) + \sum_{j=1}^{n} \chi_{\varepsilon}^{(j)}(x,y)v^{(j)}(x,y;\varepsilon) +$$

$$+ \sum_{i=1}^{n+1} \theta(r_i)w_i(\varepsilon^{-1}x_i, \varepsilon^{-1}y_i;\varepsilon) + \chi_{2,\varepsilon}(x,y)v_2(x,y;\varepsilon).$$

Thank you!