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Quantum Chaotic Scattering and Resistance Fluctuations in Mesoscopic Junctions

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Using the semiclassical approximation we calculate the quantum fluctuations of the two-dimensional electronic scattering cross-section from three hard disks in a magnetic field. The three-disk billiard is a schematic model for a three-lead mesoscopic (ballistic) junction. As the magnetic field increases the classical trajectories in the three-disk billiard undergo a hyperbolic–non-hyperbolic transition, i.e. a transition from a completely chaotic phase space to a phase space with stable islands. We show that the traces of this transition can be seen in the quantum fluctuations of the cross-section and should therefore influence the resistance fluctuations of ballistic junctions.

In the last few years a great deal of interest has been focussed on quantum manifestations of classically chaotic systems ("quantum chaos" [1, 2]). Such systems include microjunctions and other mesoscopic cavities in which the corresponding phenomena can be observed via ballistic transport coefficients. If the de-Broglie wavelength is considerably smaller than the typical length scale of the confining potential, the semiclassical formalism [1 to 3] is applicable. It has proved to be a powerful tool to understand, e.g., the spectral correlations of systems the classical dynamics of which is non-integrable and therefore exhibits chaotic behaviour. The quantum mechanical wave function can be described completely by quantities derived from the underlying classical dynamics. In particular the correlation functions of observables which exhibit quantum fluctuations are related to the statistics of the classical trajectories [4] via a simple Fourier transform. Important examples of such observables are scattering amplitudes and the corresponding intensities, since these quantities are related to the microscopic (or mesoscopic) transport coefficients by the Landauer-Büttiker formalism [2, 3].

In the present contribution we investigate quantum scattering properties of the three-disk billiard in an applied magnetic field. This system, which can be considered as the inner part of a three-lead junction has been shown [5] to exhibit a hyperbolic–non-hyperbolic transition with increasing field. It has already been shown by the present authors [6, 7] that traces of this transition can be seen in the fluctuations of the quantum scattering amplitude. Here we demonstrate, how the appearance of stable (KAM) trajectories influences the quantum fluctuations of the scattering cross-section. We argue that the resistance fluctuations of a microscopic junction therefore should also strongly be influenced by the change in the classical phase space.

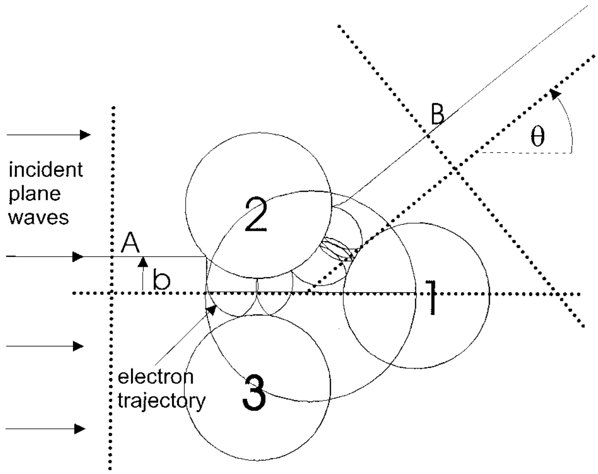

Fig. 1. Scattering geometry for the open three-disk billiard. The classical action S_j is calculated between the points A and B. The disks (radius r_{d}) are arranged such that their centers form an equilateral triangle (side length $a/r_{\mathrm{d}} = 2.5$). The magnetic field $B = \hbar k/qR$ (where R is the cyclotron radius) is assumed to be confined by a circle with radius r_{B}, which touches the corners of the triangle

We start by specifying the 2D scattering geometry (see Fig. 1) The electrons (mass m_{e}) entering the arrangement with a velocity $v = \hbar k/m_{\mathrm{e}}$ are reflected specularly by three disks (radius r_{d}) the centers of which form an equilateral triangle (side length a). The applied magnetic field B is oriented rectangularly to the plane and acts only inside the circle (radius r_{B}) which touches the midpoints of the disks. This leads to circular orbits with cyclotron radius $R \propto 1/B$ inside the scattering region. The incoming electrons are characterized by the spatial coordinate b rectangular to the current (impact parameter). The outgoing particles have a direction which differs from the incident direction by the deflection angle $\theta(b)$. This function (deflection function) completely specifies the classical scattering properties of the system. In terms of this function the classical differential scattering cross-section is given by $(\mathrm{d}\sigma/\mathrm{d}\theta)_{\mathrm{class}} = \sum_j c_j(\theta) = \sum_j |\mathrm{d}\tilde{\theta}/\mathrm{d}b_j|_{\tilde{\theta}(b_j)=\theta}$. The summation goes over all trajectories T_j which are scattered into direction θ.

As stated above, the invariant set of this chaotic scattering setup exhibits a transition from entirely chaotic (hyperbolic) behaviour to a situation where stable Kolmogorov-Arnold-Moser trajectories ("KAM-tori") are present (non-hyperbolic). For $a/r_{\mathrm{d}} = 2.5$ this transition appears at $R/r_{\mathrm{d}} \approx 0.6$.

The striking difference in the statistics of the scattering trajectories in the two cases can be best discussed with the help of the function $N(n)$. This function gives (for a given continuous range of impact parameters b) the number of trajectories which have not left the scattering region after n reflections. In terms of the quantities $c_j(\theta)$ it is given by

$$N(n) = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\theta \sum_{n_j \le n} c_j(\theta)\,, \qquad (1)$$

where n_j is the number of reflections of the trajectory T_j. This function has been calculated [6, 7] for $R/r_{\mathrm{d}} = \infty$ (hyperbolic case) and for $R/r_{\mathrm{d}} = 0.5$ (non-hyperbolic case). In the hyperbolic case this function decreases exponentially in the entire n range ($N(n) \propto \exp\{-\lambda_1 n\}$) with an escape rate $\lambda_1 = 0.48$. In the non-hyperbolic case $N(n)$ decays in a much more complicated way (see Fig. 2 of [6]). For values of n smaller than

≈ 10 the decay is nearly as rapid as in the hyperbolic case. In the regime between $n \approx 10$ and $n \approx 300$ $N(n)$ decays exponentially with a much slower rate $\lambda_2 = 0.038$. For larger n a crossover to an algebraic asymptotic decay $N(n) \propto n^{-\beta}$ with $\beta = 1.37$ is observed. This interesting behaviour can be explained as follows [6, 7]: It is known [8] that in systems with stable periodic orbits (KAM-tori) all other trajectories tend to follow these orbits for a long time. This phenomenon is known as the "stickiness" of the KAM-tori. The algebraic decay can be attributed to the existence of a Cantor set of destroyed KAM-tori ("Cantori") a the KAM surface [10, 11].

We turn now to the discussion of the quantum behaviour of the three-disk billiard. The quantum scattering cross-section is expressed in terms of the scattering amplitude $f(\theta)$ by $\mathrm{d}\sigma/\mathrm{d}\theta = |f(\theta)|^2$. In the semiclassical approximation $f(\theta)$ is given by [9]

$$f(\theta) = \sum_j (c_j)^{1/2} \exp\left\{ i \left[\frac{1}{\hbar}\, S_j + \frac{\pi}{2}\, \mu_j \right] \right\}. \tag{2}$$

Here μ is the Maslov index [9] which increases by 2 for each reflection and by 1 for each caustics encountered by the trajectory. S_j is essentially the Hamiltonian action $S_j = \int_{T_j} \mathbf{p}\, \mathrm{d}\mathbf{q} = \hbar k \tilde{L}_j$ along the classical trajectory T_j which leads through the scattering region (see [6] for details). Here we have introduced the effective length $\tilde{L}_j$ of T_j (which is equal to its actual length for $R = \infty$).

In order to discuss fluctuations we average $\mathrm{d}\sigma/\mathrm{d}\theta$ around k over a range η and subtract the result away: $\widetilde{(\mathrm{d}\sigma/\mathrm{d}\theta)}\,(k) = (\mathrm{d}\sigma/\mathrm{d}\theta)\,(k) - \langle (\mathrm{d}\sigma/\mathrm{d}\theta)\,(k)\rangle_\eta$. We then define the correlation function as

$$K_{\theta, R, k_0}(\varkappa) = \left\langle \widetilde{\frac{\mathrm{d}\sigma}{\mathrm{d}\theta}}\,(k)\, \widetilde{\frac{\mathrm{d}\sigma}{\mathrm{d}\theta}}\,(k+\varkappa) \right\rangle_{\Delta k}, \tag{3}$$

where the average is performed over a range Δk.

If one inserts $\mathrm{d}\sigma/\mathrm{d}\theta = |f(\theta)|^2$ into the expression one obtains

$$K_{\theta, R, k_0}(\varkappa) = \underbrace{\sum_{\substack{u,v \\ u \neq v}} c_u c_v [\bar{\xi}(\eta(\tilde{L}_u - \tilde{L}_v))]^2 \exp\{i\varkappa[\tilde{L}_u - \tilde{L}_v]\}}_{K^{\mathrm{d}}_{\theta, R}(\varkappa)} + \left\langle \sum_{\substack{u,v,w,z \\ u\neq v,\, w \neq z \\ u \neq z,\, w \neq v}} \cdots \right\rangle_{\Delta k}, \tag{4}$$

where $\bar{\xi}(x) \equiv 1 - \sin(x)/x$. In (4) we have divided the sum into a diagonal term $K^{\mathrm{d}}_{\theta, R}(\varkappa)$ and a non-diagonal one. The latter averages to zero for a sufficiently large value of $\Delta k/k$. Neglecting this term is called the diagonal approximation. If we now make the following assumptions, namely that (i) the c_u do not depend strongly on the angle θ, and that (ii) the effective lengths are proportional to the corresponding number of reflections (i.e. $\tilde{L}_u \approx n_u \delta$), and (iii) $\bar{\xi}(x)$ is replaced by 1; then it is clear from eq. (1) that the correlation function $K^{\mathrm{d}}_{\theta, R}(\varkappa)$ can be expressed as

$$K^{\mathrm{d}}_{\theta, R}(\varkappa) \approx \left| \sum_{n=1}^{\infty} P(n) \exp\{in\delta\varkappa\} \right|^2, \tag{5}$$

where $P(n) = N(n-1) - N(n)$ is the density which corresponds to the distribution $N(n)$, i.e. the number of trajectories which have n reflections. For $P(n) \propto \exp\{-\lambda n\}$ (hyperbolic case) one obtains [4] $K^{\mathrm{d}}_{\theta, R}(\varkappa) \propto [\varkappa^2 + (\lambda/\delta)^2]^{-1}$, i.e. a Lorentzian with a width λ/δ.

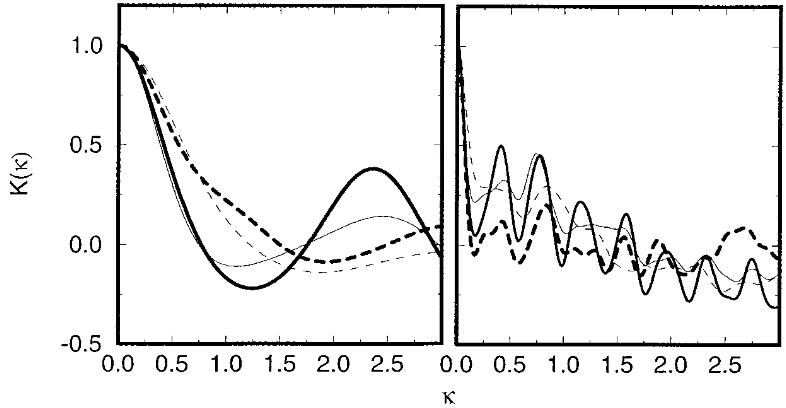

Fig. 2. Correlation functions of scattering cross-sections. Left: hyperbolic case ($R = \infty$); right: non-hyperbolic case ($R = 0.5r_\mathrm{d}$). Thick lines: Full correlation function $K(\varkappa)$, thin lines: Diagonal correlation function $K^\mathrm{d}(\varkappa)$. Continuous lines: $\theta = 3.4$, broken lines: $\theta = 5.4$. The averaging intervals are $\eta = \Delta k = 30/r_\mathrm{d}$ around $k = 1000/r_\mathrm{d}$

In Fig. 2 we have plotted $K_{\theta,R}(\varkappa)$ together with $K^\mathrm{d}_{\theta,R}(\varkappa)$ for the hyperbolic (left) and the non-hyperbolic case (right). For very small $\varkappa$ all functions coincide and have a Lorentzian shape with widths λ_1/δ and λ_2/δ, respectively. This is the regime, where approximation (5) applies. This striking difference in the initial decay indicates that in the non-hyperbolic case the fluctuations are much more correlated than in the hyperbolic one. The second striking feature is the pronounced oscillations of the correlation function in the non-hyperbolic case. These oscillations are due to the fact that standing waves are formed by the quantized long orbits. These give rise to resonances in the cross-sections due to a Bohr-Sommerfeld-like quantization of the KAM-tori.

The wavenumber dependent fluctuations can be transformed in a straightforward manner to fluctuations as a function of the Fermi level or a small-scale variation of the applied field. Since the quantum traces of the transition from a chaotic to a mixed phase space appear to be generic to microscopic junctions with rounded edges in an applied field, we are convinced that these traces must be measurable by investigation of resistance fluctuations.

References

[1] M. GUTZWILLER, Chaos in Classical and Quantum Mechanics, Springer-Verlag, New York 1991.
[2] M.-J. GIANNONI, A. VOROS, and J. ZINN-JUSTIN (Ed.), Chaos and Quantum Physics, North-Holland Publ Co. New York 1991.
[3] E. AKKERMANS, G. MONTAMBAUX, J.-L. PICHARD, and J. ZINN-JUSTIN (Ed), Mesoscopic Quantum Physics, North-Holland Publ. Co., New York 1995.
[4] R. BLÜMEL and U. SMILANSKY, Phys. Rev. Lett. **60**, 477 (1988).
[5] W. BREYMAN, Z. KOVÁCS, and T. TÉL, Phys. Rev. E **50**, 1994 (1994).
[6] M. EICHENGRÜN, W. SCHIRMACHER, and W. BREYMANN, Europhys. Lett. **36**, 483 (1996).
[7] M. EICHENGRÜN, Doctoral Dissertation, Technische Universität München, 1997, unpublished.
[8] Y. LAI, R. BLÜMEL, E. OTT, and C. GREBOGI, Phys. Rev. Lett. **68**, 3491 (1992).
Y. LAI, M. DING, C. GREBOGI, and R. BLÜMEL, Phys. Rev. A **46**, 4661 (1992).
[9] V. P. MASLOV and M. V. FEDORIUK, Semiclassical Approximation in Quantum Mechanics, Reidel, Boston 1981.
[10] J. D. MEISS and E. OTT, Phys. Rev. Lett. **25**, 2741 (1985).
[11] R. KETZMERICK, Phys. Rev. B **54**, 10841 (1996).