Shape Analysis with Subspace Symmetries

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Overview

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Related Work

Subspace Symmetries

Detection Algorithm

Results

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Partial Symmetries

Partial Symmetry Detection

- Find similar parts
- Decomposition into building blocks
- Fundamental tool in shape understanding
Partial Symmetries

Partial Symmetry Detection

Repetitive Parts
Partial Symmetries

Partial Symmetry Detection

- Repetitive part $P$ (sufficiently large)
- Transformations $f_i \in G$
- Group of transformations $G$
Restriction

- Fixed group of transformations
  - Rigid motions, reflections, scaling, affine maps
  - Intrinsic isometries
- Need to define *a priori* what constitutes similarity
More General Symmetries
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Symmetry Detection

Fixed Transformation groups

• Reflections
  [Podolak et al. 2006], [Loy et al. 2006]

• Euclidean Transformations
  [Bokeloh et al. 2009]

• Similarity transforms
  [Mitra et al. 2006], [Pauly et al. 2008]

• Intrinsic isometries
  [Ovsjanikov et al. 2008],
  [Lasowski et al. 2009], [Xu et al. 2009]
  [Mitra et al. 2010], [Kim et al. 2010]
Global Matching of General Shapes

Global Matching

• Topological Methods
  ▪ [Hilga et al. 2001]

• Combinatorial Search
  ▪ [Zhang et al. 2008], [Au et al. 2010]

• Learning
  ▪ [Kalogerakis et al. 2010],
    [van Kaik et al. 2011], [Sunkel et al. 2011]
Building subspace models

- Local matching, user guided
  - [Blanz et al. 1999], [Allen et al. 2003],
  - [Hasler et al. 2009]
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Subspace Symmetries

No transformation groups

• (Almost) arbitrary mappings
• How to avoid spurious matches?

Key idea

• Matching functions must form low dimensional subspace
Subspace Symmetries

Shapes

Subspace (1D)
Objective

Find

- Part $\mathcal{P}$
- Functions $f_1, \ldots, f_n$

Such that:

$$f_i(\mathcal{P}) = \mathbf{T} \left( \mathcal{P} + \sum_{k=1}^{d} \lambda_k b_k(\mathcal{P}) \right)$$

$d \ll n$

Rigid Motion (Param)  Shape Coordinates (Param)

Mean (Model)  Basis function (Model)
Remarks

Uniqueness:
- Many equivalent subspace models might fit the same data
- Symmetry breaking: minimize bending

Gaussian Model:
- We can learn covariance from data
- Additional constraint
Challenge

Input

- Shape $\mathcal{S} \subseteq \mathbb{R}^3$

Unknowns

- Part $\mathcal{P} \subseteq \mathcal{S}$
- Functions $f_1, ..., f_n$

Can be computed

- Rigid transformations $T_1, ..., T_n$
- Basis functions $b_1, ..., b_n$
- Shape coordinates $\lambda_1, ..., \lambda_n$
Challenge

Unknowns

- Part $\mathcal{S} \subseteq \mathbb{R}^3$
- Functions $f_1, \ldots, f_n$

Problems

- Need correspondences
- High dimensional objects
- Very large search space
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Three Steps to Reduce Complexity

1. Feature matching
   - Sparse, discrete matching

2. Graph matching
   - Matching heuristic

3. Optionally: User training
   - Learn graphs from user input
Feature Extraction

Features: surface curves & crossings

- **Strong assumption:** Graphs invariant under symmetry
- See paper technical details
Feature Matching

Brute-force feature matching

• $d$-dimensional subspace, $n$ feature points
• Brute force algorithm: double exponential in $d$

Need more efficient strategy
Heuristic Bootstrapping

Stronger Assumption
- Corresponding parts have similar feature graphs

Similar
- Same topology (small defects possible)
- Similar geometry
  - Angles, up to some noise
  - Intrinsic distances up to factor 3x

Bootstrapping
- Find a few instances first, build PCA model
- Partial finds more
Graph Matching
Complete & Partial Matches
Use discovered subspace model
Dense Correspondences

Deformable ICP

- Fit bending minimizing dense correspondence field
- Thin-plate-splines
Result
Result

initial match

principal eigenvalue
Extension: Manual Training

Training

• Click on corresponding feature points
• Mark relevant lines (one instance)
• Then: Learn PCA model of relevant graph parts

Usage

• Noisy, cluttered feature graphs
• Focus on „interesting“ subset
• Instance retrieval is still automatic
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Chairs (synthetic)
Chairs (synthetic)
Living Room (synthetic)
Living Room (synthetic)
Statue (3D Scan)
Dino (3D Scan, Manual Training)
Church (3D Scan)
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Conclusions

General notion of symmetry
- Important problem
- Proposal: subspace model

Heuristic graph matching algorithm
- Can get good results on clean input
  - Meshes and range data
  - Parameter dependent
- Training improves performance on ambiguous data

Future challenge
- Provably efficient and effective solution